

Optimal Advance Selling Strategy under Price Commitment

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Abstract

This paper considers a two-period model with experienced consumers and inexperienced consumers. The retailer determines both advance selling price and regular selling price at the beginning of the first period. I show that advance selling weekly dominates no advance selling, and the optimal advance selling price may be at a discount, at a premium or at the regular selling price. To help the retailer choose the optimal pricing strategy, conditions for each possible advance selling strategy to outperform others are characterized. Furthermore, how the consumer composition affects the retailer's optimal pricing strategy and profit are examined. In the extension, a special case with no experienced consumers provides a new explanation of advance selling price premium. That is, without experienced consumers, there are no incentives for the retailer to implement advance selling at a premium price. Besides, another special case indicates that advance selling strictly dominates no advance selling when consumer valuation distribution is uncertain. With pre-order information obtained in the first period under advance selling, the retailer is able to know the consumer valuation distribution and thus better forecast the future demand.

Key words: advance selling, price commitment, endogenous price, demand uncertainty, experienced consumers.

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1 Introduction

Advance selling is a sale strategy by a retailer which allows consumers to submit pre-orders before the release of a new product. With remarkable developments in the Internet and information technology, it is widely used in many product categories, such as books, CDs, video games, smart phones, software, fashion products, and travel services. Under advance selling, retailers offer consumers an opportunity to purchase goods or services in advance to receive guaranteed prompt delivery on the release date, which is valuable to consumers in anticipation of a future stock out. However, consumers are uncertain about their true valuations at the time of pre-ordering. To those consumers who purchase in advance, there is a risk that their true valuations are lower than what they expected.

Advance selling is commonly observed to be implemented with price commitment. At the beginning of the first period, retailers announce the advance selling price and regular selling price simultaneously and commit to the regular selling price. For example, Apple adopted advance selling for every new generation of iPhone in recent years and announced both prices on its product release conference. There are usually no discount for pre-orders. Amazon listed two prices for some coming-soon products online, such as books, video games, and music CDs, where consumers are likely to get discounts if they buy in advance. In addition, some retailers, such as Filene & Basement, Lands End and Syms, adopted advance selling at premium prices by announcing advance selling prices and future price discounts at the beginning of the first period (Zhao and Pang 2011). After observing prices for purchases in both periods, consumers make decisions to pre-order or not by comparing their expected payoffs. Before the second period starts, retailers should determine how much to stock to satisfy the total demand.

The present study on advance selling with price commitment is motivated by the following two observations.

First, for a new generation of any series product, there are a portion of consumers who had the early generation(s) of this particular product. For these experienced consumers, they are more certain about their valuations compared to other consumers without early experience. Intuitively, retailers would benefit from advance selling with the presence of experienced consumers. However, in practice, some retailers do not adopt advance selling for their new generations. When they choose advance selling, the pre-order price is either a premium price, a discount price or the same price as the regular selling season. But the literature restricts the retailer's strategy to either advance selling price premium or advance selling price discount, and thus fails to study the possibility that all four pricing strategies may be the optimal pricing strategy for the retailer in different situations.

Second, for a completely new product, including the first generation of series products, three common pricing strategies for retailers are: advance selling at a discount price; advance selling at the regular selling price; and no advance selling. Without experienced consumers in the market, it is rare that retailers choose to sell in advance at premium prices, which points to a very important role played by experienced consumers. Examining behaviors of both experienced and inexperienced consumers in the market may contribute to the literature by providing a new explanation of advance selling price premium.

The goal of this paper is to provide retailers with the guideline for the optimal pricing strategy before the release of new products. In detail, my main research questions are the following.

For a new product with early generation(s), what is the retailer's optimal pricing strategy? Will he adopt advance selling? If yes, how does the retailer choose from those possible pricing strategies? How much does he produce under each pricing strategy? With regard to a completely new product, do the answers to the above questions change? What can we learn from these changes due to the absence of experienced consumers? Finally, how does the retailer's optimal pricing strategy and profit change with the proportion of experienced consumers?

To address these research questions, I study a two-period model with both experienced consumers and inexperienced consumers. The retailer has the option to sell prior to production. He determines advance selling price and regular selling price simultaneously at the beginning of the first period if he decides to sell in advance. If not, he determines the regular selling price and the inventory before the product release. Both consumer demand uncertainty and consumer valuation uncertainty are captured in this paper by modeling that inexperienced consumers do not know their valuations until the regular selling season and their number is unknown. Furthermore, it is assumed that consumer valuation follows a two-point distribution.

The main findings, summarized in the last section, indicate that the retailer has four possible pricing strategies: advance selling at a premium price, advance selling at a discount price, advance selling at the regular selling price, and no advance selling. The conditions for each possible pricing strategy to prevail are also characterized. Furthermore, we see that the presence of experienced consumers is the emotivity for the retailer to adopt advance selling at a premium price. However, the retailer will always choose advance selling when he is uncertain about the consumer valuation distribution. What he learns from pre-orders helps to pinpoint consumer valuation distribution and thus improve his forecast of future demand.

This paper contributes to a growing literature on advance selling under price commitment that allows the regular selling price to be chosen endogenously rather than taking it as an exogenous parameter in other closely related papers (Zhao and Stecke 2010, Prasad, Stecke, and Zhao 2011). I believe that how much to price in the regular selling season is vital to retailers as well when they decides to sell in advance or not; and an endogenously determined regular selling price will bring more interesting results to the literature.

Secondly, discovering the prevalence of advance selling at the regular selling price in the equilibrium and describing the conditions for it are also of great significance to the literature. While all other studies focus on either advance selling price discount or advance selling price premium, this current study firstly concludes that advance selling at the regular selling price may be the optimal strategy for the retailer. In detail, it describes the situation that all four possible pricing strategies may arise in the equilibrium, and compares these strategies to explore when each strategy is optional and how each pricing strategy is implemented.

Moreover, this paper is one of the few in the literature to model the market segments in an operational context by dividing consumers into two groups: experienced and inexperienced (Loginova, Wang and Zeng 2011, 2012). After comparing with a model without experienced consumers, this paper further studies the role played by experienced consumers in the market and provides prescriptive insights for a retailer's decisions.

After the literature review, the rest of the paper is organized as follows. Section 3 introduces the model. Section 4 focuses on advance selling and presents the analysis of optimal advance selling strategies. Section 5 compares advance selling with no advance selling and generates the optimal pricing strategy for the retailer. Furthermore, a robust comparison is conducted to

show how the retailer chooses from these pricing strategies. Section 6 examines how consumer composition affects the retailer's optimal strategy and profit. Section 7 studies two special cases. Section 8 concludes the paper.

2 Literature Review

As a newly emerging sale strategy, advance selling is not only widely adopted in service industry (Xie and Shugan 2001, Möller and Waternabe 2010, Sainam, Balasubramanian, and Bayus 2010), but also playing an increasingly important role in manufacturing industry (Moe and Fader 2002, Hui, Eliashberg, and George 2008, McCardle, Rajaram, and Tang 2004).

The literature points out three major benefits associated with advance selling. First, it helps the retailer to reduce the demand uncertainty (e.g., Boyaci and Özer 2011, Loginova, Wang and Zeng 2011). Second, it provides the retailer with opportunities to better forecast the future demand (e.g., Tang, Rajaram, Alptekinoglu, and Ou 2004, Prasad, Stecke, and Zhao 2011). Third, it helps the retailer to utilize consumers' uncertainty of valuations and increase the overall demand (e.g., Xie and Shugan 2001, Zhao and Stecke 2010).

This paper is closely related to the literature on advance selling under price commitment in manufacturing industry. Weng and Parlar (1999) are the first to develop a model in which pre-orders are offered with a discount to attract consumers. Tang, Rajaram, Alptekinoglu, and Ou (2004) extend the model by Weng and Parlar (1999) and examine the benefits of advance selling. Chen and Parlar (2005) introduce two different models and solve for the optimal advance selling discount and optimal quantity. McCardle, Rajaram, and Tang (2004) present a duopoly model and focus on competition between two firms. In these four papers discussed above, consumers are modeled to be non-strategic. Other papers assume strategic consumers and incorporates consumers' decision-making process into retailers' consideration. Optimal advance selling strategies are examined in different settings. For examples, Zhao and Stecke (2010) classify consumers into two groups according to whether they are loss averse and examine the optimal advance selling strategy for the retailer while Prasad, Stecke, and Zhao (2011) divide consumers into two groups, informed consumers and uninformed consumers, based on the accessibility to the pre-order information. Loginova, Wang and Zeng (2011) divide consumers into two groups, experienced and inexperienced, and study advance selling premium together with advance selling discount.

Most studies in this group treat the regular selling price as exogenously given and study the optimal advance selling price for the retailer. The only exceptions are Chu and Zhang (2011), Nocke, Peitz, and Rosar (2011), Zhao and Pang (2011) and Nasiry and Popescu (2012), where the seller decides and announces both advance selling price and regular selling price at the beginning of the first period. In Chu and Zhang (2011) and Nocke, Peitz, and Rosar (2011), population size is normalized to 1 and therefore there is no aggregate demand uncertainty. There is always an pre-order discount to induce consumers to purchase in advance. Zhao and Pang (2011) assume that the retailer will adopt advance selling and compare price commitment with two other pricing strategies: dynamic pricing and pre-order price guarantee. Both markdown and markup may be possible under price commitment. Nasiry and Popescu (2012) study the seller's optimal advance selling strategy in a context of consumers' regret. However, none of

these papers explain that why there are some firms in practice adopt advance selling at the regular selling price.

Other than study advance selling under price commitment, Li and Zhang (2010) takes regular selling price as a decision variable and analyzes dynamic pricing strategy. In their paper, the seller sets advance selling price first, and chooses regular selling price after pre-orders are placed. They conclude that advance selling discounts are not possible. In Shugan and Xie (2004), the seller announces the spot and advance prices in the first period, and the spot price in the second period. Compared to spot-only strategies, this paper concludes that advance selling may increase the seller's profit. Furthermore, capacity constraints allow the seller to charge premium for pre-orders.

Another related body of literature assumes uncertain valuations for consumers. Most of the studies above on advance selling fall into this category. Besides, Akan, Ata, and Dana (2007) study a model with heterogeneous consumers who privately learn their true valuations in the future. They show that the optimal mechanism is a menu of refund contracts with various expiration dates. Liu and Xiao (2008) assume that consumers know their intrinsic value of the product before purchases but are uncertain about the fitness. The firm's optimal return policy is examined under this setting. Swinney (2011) considers a retailer selling to a strategic customer population with uncertain valuations and examines the quick response regime. Loginova, Wang and Zeng (2012) build up a model with a group of consumers who know their valuations from the beginning of the first period while all other consumers do not to study the retailer's advance selling strategy with learning from pre-orders.

It is commonly observed that there are four possible policies at optimality: advance selling at a premium price; advance selling at a discount price; advance selling at the regular selling price and no advance selling. All of the aforementioned papers on advance selling have their limitations to explain the practice of advance selling in the real world. Specifically, most studies restrict the seller's strategy to advance selling price discount (e.g., Zhao and Stecke 2010, Prasad, Stecke, and Zhao 2011, Chu and Zhang 2011 and Nocke, Peitz, and Rosar 2011), while some of the others study advance selling price premium and provide the condition for it to prevail (e.g., Li and Zhang 2010, Zhao and Pang 2011, Loginova, Wang and Zeng 2011, 2012, and Nasiry and Popescu 2012). However, none of the literature captures the fact that advance selling at the regular selling price might also arise in the equilibrium and provides retailers with the guideline to choose from these four common-observed policies.

To the best of my knowledge, this is the first paper to show how retailers implement one of the above four strategies under price commitment in a context with (a) endogenously determined regular selling price ; (b) experienced and inexperienced consumers; (c) demand uncertainty and consumer valuation uncertainty; and (d) endogenously determined stock-out probability. Furthermore, it explores the role played by experienced consumers in the market and provides a new explanation for why the retailer charges premium for pre-orders. But more than that, it examines how the consumer composition and the uncertainty in consumer valuation distribution affect the retailer's optimal pricing strategy.

3 Model Setup

Consider a retailer (“he”) produces a new product at marginal cost c and sells it to the market over two periods. The first period is the advance selling season and the second period is the regular selling season. Each consumer wants to purchase at most one unit of the specific product.

The retailer chooses price commitment strategy. He will commit to the selling price at the beginning of the advance selling season. Let p_1 and p_2 denote advance selling price and regular selling price, respectively. At the beginning of the first period, both prices are decided by the retailer and announced simultaneously to consumers. While pre-orders are guaranteed to be fulfilled right after the product release, orders in the second period face a risk of stock-out. For each product unsold at the end of the second period, the retailer gets a salvage price s . It is assumed that $s < c < p_i, i = 1, 2$.

There are two types of consumers, experienced and inexperienced, depending on whether they had experience of early generations of this product. The number of experienced consumers, m_e , is known. The number of inexperienced consumers, M_i , is a random variable, which follows lognormal distribution $\text{LN}(\nu_i, \tau_i^2)$. Let m_i denote the expected value (mean) of M_i , where $m_i = \exp\{\nu_i + \tau_i^2/2\}$.

Following Xie and Shugan (2001), the consumer valuation of this product V is assumed to follow a Bernoulli distribution, with probability k to realize a high valuation v_H and probability $1 - k$ to realize a low valuation v_L , where $v_L < v_H$ and $0 < k < 1$.¹ Experienced consumers know their valuations from the beginning of the first period. However, inexperienced consumers realize their valuations after the product release. Table 1 lists the notation in this paper. The market behaviors can be described in a two-period process:

1. At the beginning of the advance selling season the retailer decides on the advance selling price p_1 and regular selling price p_2 . In this period, experienced consumers know their valuations, but inexperienced consumers do not. Each consumer observes both prices and makes the decision to pre-order or not.

At the end of the advance selling period the retailer gets the number of pre-orders, denoted by D_1 . Informed by the pre-orders, the retailer must decide how much to produce: $Q = D_1 + q$, where D_1 fulfills the pre-orders and quantity q satisfies the stochastic demand during the regular selling season, denoted by D_2 .

2. After the regular selling season starts, inexperienced consumers know their valuations. For experienced consumers and inexperienced consumers who did not pre-order, they make purchases if their valuations exceeds the regular selling price p_2 . All pre-orders will be delivered to consumers right after the release date. With regard to orders submitted in the second period, they are fulfilled depending on the stock.

¹Consumers with valuation v_L will be referred to as low type consumers hereinafter, while consumers with valuation v_H will be referred to as high type consumers.

Table 1: Notation

| Parameters/Variables concerning a retailer | |
|--|---|
| c | marginal cost |
| s | salvage value |
| π | retailer's expected profit from uncertain demand in the second period |
| Π | retailer's total expected profit (includes pre-orders) |
| Parameters/Variables concerning consumers and market | |
| D_1, D_2 | demands in the first and second periods |
| $V \in \{v_H, v_L\}$ | Bernoulli distribution, $\text{Prob}(v_H) = k$ and $\text{Prob}(v_L) = 1 - k$ |
| m_e | number of experienced consumers |
| $M_i \sim \text{LN}(\nu_i, \tau_i^2)$ | number of inexperienced consumers, mean $m_i = \exp\{\nu_i + \tau_i^2/2\}$ |
| η | stock-out probability |
| Decision variables | |
| q | quantity produced for D_2 |
| Q | total quantity produced (includes pre-orders) |
| p_1 | advance selling price |
| p_2 | regular selling price |

4 Analysis of Advance selling

In this section, the optimal advance selling strategy will be studied assuming the retailer will adopt advance selling. Specifically, backward induction is applied in Section 4.1 and 4.2 to obtain the optimal prices in both periods. Results are combined in Section 4.3 to form the optimal advance selling strategy.

4.1 The regular selling season

After the regular selling season starts, inexperienced consumers realize their valuations denoted by v . For both experienced consumers and inexperienced consumers, v can be either v_L or v_H . Any consumer who did not pre-order decides to buy if and only if $v \geq p_2$.

The retailer sets the regular selling price p_2 to maximize his total expected profit in the second period. To consumers who did not pre-order, when $p_2 \leq v_L$, all buy in the selling season and $p_2 = v_L$ yields the highest profit margin; when $v_L < p_2 \leq v_H$, only consumers with high valuations buy and $p_2 = v_H$ yields the highest profit margin; when $p_2 > v_H$, no consumers buy. Therefore, the retailer will always charge $p_2^* = v_L$ or $p_2^* = v_H$.

Lemma 1. *The optimal price in the regular selling season, p_2^* , is either v_L or v_H .*

When there are experienced consumers remaining in the market, the total expected profit

from them is very easy to calculate since m_e is known. However, when there are inexperienced consumers remaining in the market, the retailer faces uncertain demand because the number of inexperienced consumers M_i is unknown. The total expected profit from inexperienced consumers in the regular selling season is

$$\pi(q) = p_2^* \mathbb{E} [\min \{q, D_2\}] + s \mathbb{E} [(q - D_2)^+] - cq,$$

where D_2 in the above equation denotes the demand from inexperienced consumers.

The retailer chooses q to maximize his total expected profit, $\pi(q)$. Let q^* denote the optimal production quantity. When $p_2^* = v_L$, all inexperienced consumers buy in the selling season. Thus, $D_2 = M_i \sim \text{LN}(\nu_i, \tau_i^2)$. The optimal quantity for this lognormal distribution and the resulting total expected profit are calculated in Loginova, Wang and Zeng (2011). They are

$$\begin{cases} q_L^* &= \exp(\nu_i + \tau_i z_\beta), \\ \pi_L &= (v_L - s)(1 - \Phi(\tau_i - z_\beta))m_i. \end{cases} \quad (1)$$

When $p_2^* = v_H$, only inexperienced consumers with $v = v_H$ buy in the selling season. Thus, $D_2 = kM_i \sim \text{LN}(\nu_i + \ln k, \tau_i^2)$. Similarly, the optimal quantity and the resulting total expected profit are given by

$$\begin{cases} q_H^* &= k \exp(\nu_i + \tau_i z_\beta), \\ \pi_H &= k(v_H - s)(1 - \Phi(\tau_i - z_\beta))m_i. \end{cases} \quad (2)$$

In (1) and (2), $\beta = \frac{p_2^* - s}{p_2^* - c}$, $z_\beta = \Phi^{-1}(\beta)$. It is important to note that β and z_β take different values in these two equations because p_2^* are different.

4.2 The advance selling season

At the beginning of the advance selling season, the retailer announces both advance selling price and regular selling price. All experienced consumers know their valuations while all inexperienced consumer do not. Then consumers pre-order in this period if and only if the expected payoffs from pre-ordering exceeds the expected payoffs from waiting. We study the optimal advance selling price in two parts when p_2^* takes different values.

First, what is the optimal advance selling price p_1 when $p_2^* = v_L$? It is obvious that the retailer will never set $p_1 > v_H$ because no consumers will pre-order in this case. Since the retailer always incorporates consumers' purchasing behaviors into consideration before his decisions, we first take a look at consumers' optimal purchasing decisions.

Inexperienced consumers pre-order in the first period if and only if the expected payoffs from pre-ordering exceeds the expected payoffs from waiting. That is, $EV - p_1 \geq (1 - \eta)(EV - v_L)$, where η is the stock-out probability. Thus, we can get the threshold value of advance selling price which attracts inexperienced consumers to pre-order, which is denoted by

$$\hat{p} \equiv EV - (1 - \eta)(EV - v_L). \quad (3)$$

Experienced consumers with low valuation will never pre-order at $p_1 > v_L$. With regard to high type experienced consumers, they may be attracted to pre-order since there is a probability

that this product will be out of stock in the second period. If they pre-order, the payoffs are $v_H - p_1$; if they wait to buy in the selling season, their payoffs are $(1 - \eta)(v_H - v_L)$. For these experienced consumers, they pre-order if and only if $v_H - p_1 \geq (1 - \eta)(v_H - v_L)$. Thus, we can get the threshold value of advance selling price which attracts high type experienced consumers to pre-order, which is denoted by

$$\bar{p} \equiv v_H - (1 - \eta)(v_H - v_L). \quad (4)$$

Following Loginova, Wang and Zeng (2011), the stock-out probability is endogenously decided by $\eta = \mathbb{E}\left[\left(\frac{D_2 - q^*}{D_2}\right)^+\right]$, where D_2 is the stochastic demand in the second period and q^* is the optimal production quantity for D_2 . It is important to note that η takes the same value in both equations, (3) and (4). In detail, $\eta = \mathbb{E}\left[\left(\frac{D_2 - q^*}{D_2}\right)^+\right]$, where $D_2 = m_e + M_i$ and $q^* = m_e + q_L^*$. Thus,

$$\eta = \mathbb{E}\left[\left(\frac{M_i - q_L^*}{m_e + M_i}\right)^+\right]. \quad (5)$$

Lemma 2. *The threshold values \hat{p} and \bar{p} satisfy that $v_L < \hat{p} < \bar{p} < v_H$.*

In the case that $p_1 \leq \hat{p}$, all inexperienced consumers pre-order at p_1 . The profit from inexperienced consumers is $(p_1 - c)m_i$. Since $p_2^* = v_L$, all experienced consumers buy this product, either in the first period or in the second period. The profit from experienced consumers is $\begin{cases} (p_1 - c)m_e, & p_1 \leq v_L, \\ (v_L - c)m_e, & v_L < p_1 \leq \hat{p}. \end{cases}$ Therefore, the total expected profit is a nondecreasing function of p_1 . The retailer will adopt advance selling at $p_1 = \hat{p}$. In this situation, all inexperienced consumers pre-order and all experienced consumers wait to buy in the selling season. His optimal total production and resulting total expected profit are

$$\begin{cases} Q(\hat{p}, v_L) &= D_1 + m_e, \\ \Pi(\hat{p}, v_L) &= (v_L - c)m_e + (\hat{p} - c)m_i. \end{cases} \quad (6)$$

In the case that $\hat{p} < p_1 \leq v_H$, all inexperienced consumers and low valuation experienced consumers wait to buy in the second period. The retailer will choose $p_1 = \bar{p}$ because that (a) prices in (\hat{p}, \bar{p}) result in lower profit margins and (b) prices in $(\bar{p}, v_H]$ result in no pre-orders. In this situation, experienced consumers with valuation v_H pre-order. All other consumers wait to buy in the second period. The resulting demand in the first period is $D_1 = km_e$. The demand in the second period are composed of two parts. The first part comes from low valuation experienced consumers with number $(1 - k)m_e$; the second part comes from inexperienced consumers with a random number M_i . Together with (1), the optimal quantity decision and the resulting total expected profit at $p_1 = \bar{p}$ are given by

$$\begin{cases} Q(\bar{p}, v_L) &= D_1 + (1 - k)m_e + q_L^*, \\ \Pi(\bar{p}, v_L) &= (\bar{p} - c)km_e + (v_L - c)(1 - k)m_e + \pi_L. \end{cases} \quad (7)$$

Let $m = m_e + m_i$ denote the total expected number of consumers and let α denote the proportion of experienced consumers. Thus, $m_e = \alpha m$ and $m_i = (1 - \alpha)m$. For simplification, we will assume $\alpha \leq \bar{\alpha}$, where $\bar{\alpha}$ is endogenously determined in the model.

Assumption 1. *The proportion of experienced consumers in the market α satisfies that*

$$\alpha \leq \bar{\alpha} \equiv \frac{\eta(v_H - v_L) + v_L - c - (v_L - s)(1 - \Phi(\tau_i - z_\beta))}{2\eta(v_H - v_L) + v_L - c - (v_L - s)(1 - \Phi(\tau_i - z_\beta))}. \quad 2$$

Obviously, $1/2 < \bar{\alpha} < 1$, and $\bar{\alpha}$ is very close to 1 as $\eta(v_H - v_L)$ is comparatively small.³ This assumption is reasonable here which essentially means that there exists a threshold value for the proportion of experienced consumers in the market. As a result, it is probably not profitable for the retailer to set a higher premium price, $p_1 = \bar{p}$, to extract more profits from high type experienced consumers.

Lemma 3. *Under Assumption 1, advance selling at \bar{p} is strictly dominated by advance selling at \hat{p} . Therefore, when $p_2^* = v_L$, the retailer will always set the advance selling price at \hat{p} .*

Lemma 3 implies that if the proportion of experienced consumers in the market is below the threshold value $\bar{\alpha}$, there are not too many incentives for the retailer to set a higher premium, $p_1 = \bar{p}$, in order to earn extra profits from high type consumers. Therefore, the optimal advance selling price is always \hat{p} .

Then, we are going to study the optimal advance selling price p_1 when $p_2^* = v_H$. Inexperienced consumers pre-order in the first period if and only if $EV - p_1 \geq 0$. With $p_1 \leq EV$, all inexperienced consumers pre-order. Otherwise, all wait until the second period.

In the case that $p_1 \leq v_L$, all consumers pre-order and the maximum total expected profit realizes at $p_1 = v_L$. Thus, the optimal quantity decision and the resulting total expected profit at $p_1 = v_L$ are given by

$$\begin{cases} Q(v_L, v_H) &= D_1, \\ \Pi(v_L, v_H) &= (v_L - c)m_e + (v_L - c)m_i. \end{cases} \quad (8)$$

If $v_L < p_1 \leq EV$, all inexperienced consumers pre-order. Experienced consumer with $v = v_L$ do not buy this product while those with $v = v_H$ pre-order in the first period. The maximum total expected profit realizes at $p_1 = EV$. Thus, we have

$$\begin{cases} Q(EV, v_H) &= D_1, \\ \Pi(EV, v_H) &= (EV - c)km_e + (EV - c)m_i. \end{cases} \quad (9)$$

If $EV < p_1 \leq v_H$, all inexperienced consumers wait to make the decisions in the second period. Experienced consumer with $v = v_L$ do not buy this product. With regard to experienced consumers with $v = v_H$, they pre-order in the first period. The maximum expected total profit realizes at $p_1 = v_H$. Together with (2), we have

$$\begin{cases} Q(v_H, v_H) &= D_1 + q_H^*, \\ \Pi(v_H, v_H) &= (v_H - c)km_e + \pi_H. \end{cases} \quad (10)$$

²This assumption is useful because it simplifies the analysis in Section 5. The main conclusions of this paper still hold even if α is greater than $\bar{\alpha}$. The only difference is that there may be two levels of premium in the first period.

³The average value of $\bar{\alpha}$ in the numerical examples conducted is 0.85. Refer to Section 5.3.

With the trend that p_1 increases from v_L to EV and to v_H , the retailer faces a tradeoff between high price-low sales and low price-high sales. Specifically, when the retailer charges a deep discount v_L in the first period, all consumers pre-order it; However, the profit margin is low. When there is a moderate discount EV , the profit margin increases, but he loses low type experienced consumers. When there is no discount for pre-orders, the profit margin from high type experienced consumers reaches the highest; however, the retailer continue to lose consumers. Low type inexperienced consumer will not buy in the second period. Therefore, we have the following lemma.

Lemma 4. *When $p_2^* = v_H$, the retailer will set advance selling price at either v_L , EV or v_H .*

4.3 Optimal advance selling strategy

Assuming that the retailer will adopt advance selling, his optimal advance selling strategy is analyzed by combining the separate analyses in the advance selling season. With Lemma 3 and Lemma 4, there are four possible advance selling strategies: (\hat{p}, v_L) , (v_L, v_H) , (EV, v_H) and (v_H, v_H) . However, by comparing equation (6) and (8), it is obvious that (v_L, v_H) is strictly dominated by (\hat{p}, v_L) .

Proposition 1. *If the retailer decides to adopt advance selling, the optimal advance selling strategy can be any of the following: (i) advance selling at a premium with strategy (\hat{p}, v_L) ; (ii) advance selling at a discount with strategy (EV, v_H) ; (iii) advance selling at the regular selling price with strategy (v_H, v_H) .⁴*

At advance selling price premium with strategy (\hat{p}, v_L) , all inexperienced consumers pre-order at \hat{p} and all experienced wait to buy in the second period at v_L ; the retailer thus produces $D_1 + m_e$ units of product. At advance selling price discount with strategy (EV, v_H) , all inexperienced consumers and high type experienced consumers pre-order at EV ; the retailer thus produces D_1 units of product. Last, at (v_H, v_H) , high type experienced consumers pre-order at v_H and high type inexperienced consumers buy in the second period; the retailer thus produces $D_1 + q_H^*$ units of product.

Since the number of inexperienced consumers is unknown and that $\hat{p} < EV < v_H$, the three likely optimal selling strategies reflect two different tradeoffs for the retailer: between low price-high sales and high price-low sales, and between low price-low uncertainty and high price-high uncertainty. For example, both tradeoffs described above are present between (\hat{p}, v_L) and (v_H, v_H) . First, (\hat{p}, v_L) corresponds to higher sales and lower prices, while (v_H, v_H) corresponds to much lower sales and higher prices. Second, (\hat{p}, v_L) means no uncertainty in the second period, while (v_H, v_H) leads to a positive uncertainty. Comparing all three possible optimal selling strategies, we conclude that, as the strategies changes from (\hat{p}, v_L) to (EV, v_H) to (v_H, v_H) , the expected sales decrease and the demand uncertainty increases.

⁴The conditions for each possible advance selling strategy to prevail are characterized in Proposition 3.

5 Optimal pricing strategy

In Section 5.1, the optimal pricing strategy will be studied for the retailer after comparing advance selling with no advance selling. Moreover, Section 5.2 compares all possible pricing strategies and identify the conditions for each strategy to prevail. Examples are presented to demonstrate the pricing decision in Section 5.3.

5.1 Advance selling vs. no advance selling

Without advance selling, the retailer chooses the optimal regular selling price and production quantity at the beginning of the selling season. There are two types of consumers: one with a high valuation v_H , and the other with a low valuation v_L . All consumers realize their valuations in this period. The market size is $m_e + M_i$, where M_i is a random variable. As discussed before, the optimal selling price for the retailer is either v_L or v_H . The resulting expected total profits are $\Pi(v_L) = (v_L - c)m_e + (v_L - s)(1 - \Phi(\tau_i - z_\beta))m_i$ and $\Pi(v_H) = (v_H - c)km_e + k(v_H - s)(1 - \Phi(\tau_i - z_\beta))m_i$, respectively.

When the retailer adopts advance selling, his maximum expected total profit, denoted by Π^* , is realized as $\Pi^* = \max\{\Pi(\hat{p}, v_L), \Pi(EV, v_H), \Pi(v_H, v_H)\}$. It is obvious that $\Pi(v_L) < \Pi(\hat{p}, v_L) \leq \Pi^*$. Thus, the retailer will never sell in the regular selling season at price v_L without advance selling. With regard to selling in the regular selling season at price v_H , we have $\Pi(v_H) = \Pi(v_H, v_H)$. That is, selling at v_H (without advance selling) yields the same amount of profit as that from advance selling at (v_H, v_H) . As a result, it might be an option for the retailer to sell at v_H in the case that $\Pi^* = \Pi(v_H, v_H)$. Otherwise, selling at either v_L or v_H without advance selling is strictly dominated.

Proposition 2. *There are four possible pricing strategies for the retailer at optimality: advance selling at a premium price, (\bar{p}, v_L) ; advance selling at a discount price, (EV, v_H) ; advance selling at the regular selling price, (v_H, v_H) and no advance selling with strategy v_H .⁵ If $\Pi(v_H, v_H) = \Pi^*$, the retailer chooses advance selling at the regular selling price, or no advance selling. Otherwise, the retailer will always adopt advance selling, either at a premium price, or at a discount price.*

Proposition 2 implies that advance selling weekly dominates no advance selling for a retailer who will release a new product in the coming future. Thus, he chooses from these four pricing strategies. Examples in the real world perfectly support results in Proposition 2. One common sale strategy of new products is advance selling at a discount price. Consumers are often offered with discounts if they pre-order books, CDs and video games on websites like Amazon.com and Bestbuy.com. There are also several examples of advance selling at a premium price, including Filene & Basement, Lands End and Syms. These sellers announced their prices and future discounts for some products. In addition, both advance selling at the selling price and no advance selling arise in practice. For examples, Apple allowed consumers to pre-order iPhone 4, iPhone 4S and iPhone 5 at the selling price; but for the second generation (iPhone 3G) with experienced consumers in the market, it did not implement advance selling. Same for Harry Potter books

⁵For the following analysis, v_H is used to describe the strategy that the retailer sells in the second period at v_H without advance selling.

which appear as a popular example of advance selling. Pre-orders were not available for the second book of Harry Potter, but it became available for later ones on Amazon.com.

Since there is a tie between these two pricing strategies, v_H and (v_H, v_H) , it needs the retailer to weigh the possible trade-offs in practice to make the decision when $\Pi^* = \Pi(v_H, v_H)$. Although it is not the focus of this paper, the reason for the retailer to choose (v_H, v_H) over no advance selling may be as follows. For some highly sought-after products, not much extra costs are needed to inform consumers about the pre-order availability. But advance selling helps to relief the crazy order traffic after the release and maintain an efficient working style. With regard to the case that the retailer chooses selling at v_H without advance selling, it may be because that the cost to implement advance selling is comparatively high.

5.2 Comparison of optimal pricing strategies

Proposition 2 talks about the four possible optimal pricing strategies in the equilibrium. The retailer chooses advance selling at the regular selling price or no advance selling if $\Pi(v_H, v_H) = \Pi^*$; Otherwise, he always chooses advance selling, either at a premium price, or at a discount price. So the next question is, when does the retailer adopt advance selling at a premium? at a discount? or at the regular selling price?⁶

To see that, rewrite $\Pi(\hat{p}, v_L)$, $\Pi(EV, v_H)$ and $\Pi(v_H, v_H)$ as functions of k and denote them by $\Pi_{premium}(k)$, $\Pi_{discount}(k)$ and $\Pi_{regular}(k)$, respectively. As shown in Appendix, we have

$$\Pi_{premium}(k) = \eta(v_H - v_L)m_i k + (v_L - c)(m_e + m_i); \quad (11)$$

$$\Pi_{discount}(k) = (v_H - v_L)m_e k^2 + \{(v_H - v_L)m_i + (v_L - c)m_e\}k + (v_L - c)m_i; \quad (12)$$

$$\Pi_{regular}(k) = \{(v_H - c)m_e + (v_H - s)(1 - \Phi(\tau_i - z_\beta))m_i\}k. \quad (13)$$

Lemma 5. *The above expected total profit functions have the following properties:*

- (i) $\frac{\partial \Pi_{premium}(k)}{\partial k} > 0$; $\frac{\partial \Pi_{discount}(k)}{\partial k} > 0$; and $\frac{\partial \Pi_{regular}(k)}{\partial k} > 0$;
- (ii) $\lim_{k \rightarrow 0} \Pi_{premium}(k) > \lim_{k \rightarrow 0} \Pi_{discount}(k) > \lim_{k \rightarrow 0} \Pi_{regular}(k)$;
- (iii) $\lim_{k \rightarrow 1} \Pi_{discount}(k) > \lim_{k \rightarrow 1} \Pi_{premium}(k)$, $\lim_{k \rightarrow 1} \Pi_{regular}(k)$.

Lemma 5(i) implies that retailer's expected total profit increases when consumers are more likely to realize high valuations, no matter which strategy he chooses. That is, the retailer is happier when there are more high valuation consumers. Lemma 5(ii) implies that if consumers are very likely to realize low valuations, the retailer will choose advance selling price premium with strategy (\hat{p}, v_L) over other strategies. Lemma 5(iii) implies that if consumers are very likely to realize high valuations, the retailer will prefer advance selling at a discount price. However, if consumer valuation uncertainty is high, it is not clear which pricing strategy will outperform others. Would it be profitable for the retailer to choose advance selling at the regular selling price with strategy (v_H, v_H) ?

Figure 1 shows four possible patterns in the interval $k \in (0, 1)$ for these profit functions.

⁶The same conditions are needed for the retailer to sell in the second period at v_H without advance selling.

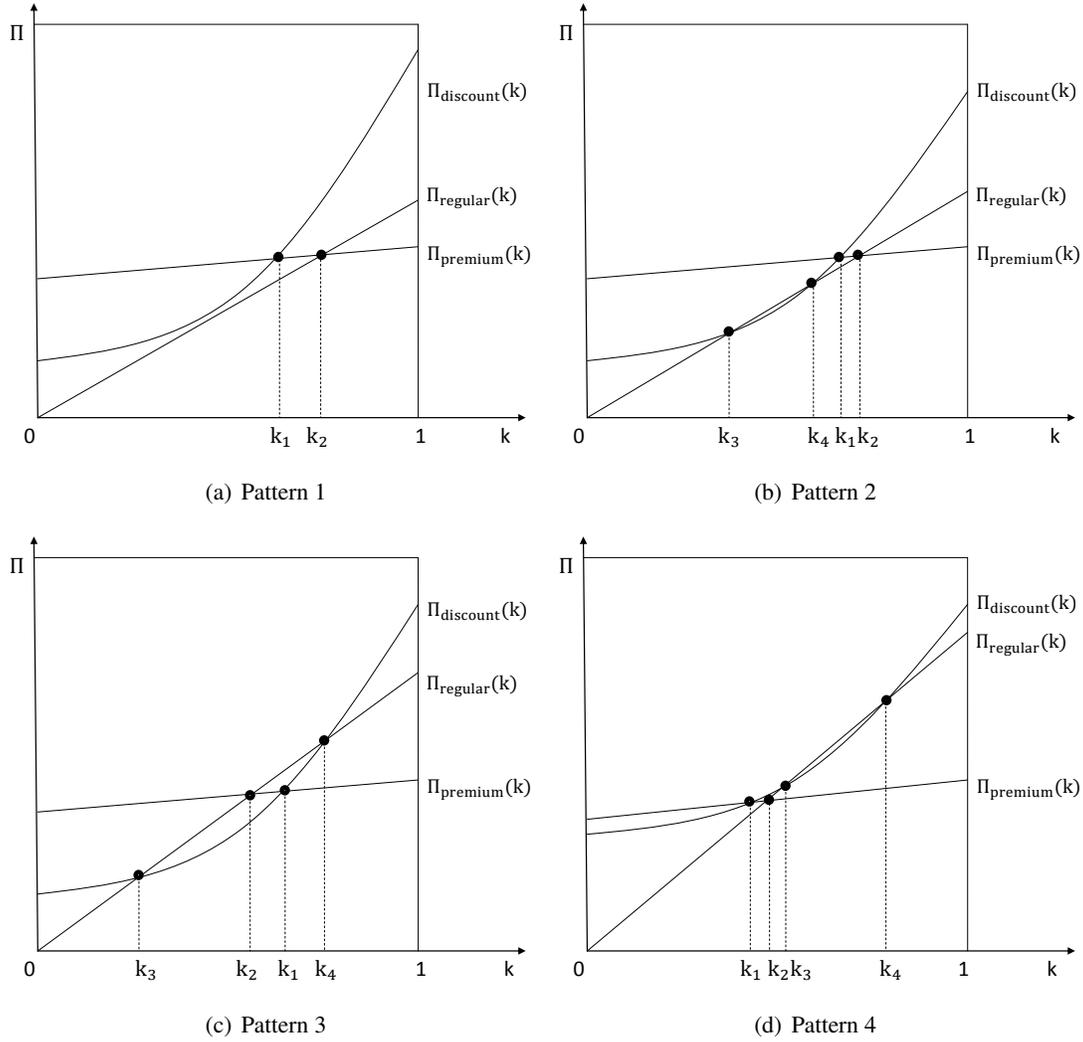


Figure 1: Optimal advance selling price⁷

- Pattern 1: $\Pi_{discount}(k)$ intersects $\Pi_{regular}(k)$ at most once.
- Pattern 2: $\Pi_{discount}(k)$ intersects $\Pi_{regular}(k)$ twice before it intersects $\Pi_{premium}(k)$.
- Pattern 3: $\Pi_{discount}(k)$ intersects $\Pi_{regular}(k)$ once before it intersects $\Pi_{premium}(k)$ and once after it.
- Pattern 4: $\Pi_{discount}(k)$ intersects $\Pi_{regular}(k)$ twice after it intersects $\Pi_{premium}(k)$.

⁷Pattern 1 and pattern 2 include the situation that $\Pi_{regular}(k)$ is entirely below $\Pi_{premium}(k)$.

In Figure 1, $\Pi_{premium}(k)$ intersects $\Pi_{discount}(k)$ once in the interval $k \in (0, 1)$ (implied by Lemma 5). Let k_1 denote the intersection of $\Pi_{discount}(k)$ and $\Pi_{premium}(k)$. We have $k_1 = \frac{\sqrt{\zeta^2 + 4(v_H - v_L)(v_L - c)m_e^2} - \zeta}{2(v_H - v_L)m_e}$, where $\zeta = (1 - \eta)(v_H - v_L)m_i + (v_L - c)m_e$.

However, the intersection of $\Pi_{regular}(k)$ and $\Pi_{premium}(k)$, denoted by k_2 , may not locate in the interval $k \in (0, 1)$. We can calculate k_2 as $\frac{(v_L - c)(m_e + m_i)}{(v_H - c)m_e + (v_H - s)(1 - \Phi(\tau_i - z_\beta))m_i - \eta(v_H - v_L)m_i}$.

Lemma 6. k_2 locates in $(0, 1)$ if the following condition holds:

$$(v_H - v_L)m_e > \left(\eta(v_H - v_L) + v_L - c - (v_H - s)(1 - \Phi(\tau_i - z_\beta)) \right) m_i. \quad (14)$$

Furthermore, $\Pi_{regular}(k)$ may not intersect $\Pi_{discount}(k)$ in the interval $k \in (0, 1)$. Let k_3 and k_4 denote the intersections of $\Pi_{regular}(k)$ and $\Pi_{discount}(k)$ if they exist (assuming that $k_3 < k_4$). We can obtain that $k_3 = \frac{-\sqrt{\xi^2 - 4(v_H - v_L)m_e(v_L - c)m_i} - \xi}{2(v_H - v_L)m_e}$, and $k_4 = \frac{\sqrt{\xi^2 - 4(v_H - v_L)m_e(v_L - c)m_i} - \xi}{2(v_H - v_L)m_e}$, where $\xi = (v_H - v_L)(m_i - m_e) - (v_H - s)(1 - \Phi(\tau_i - z_\beta))m_i$ (see derivations for k_1, k_2, k_3 and k_4 in Appendix).

Lemma 7. $k_3, k_4 \in (0, 1)$ if the following conditions hold:

$$\begin{cases} \xi^2 - 4(v_H - v_L)m_e(v_L - c)m_i > 0, \\ 0 < -\frac{\xi}{2(v_H - v_L)m_e} < 1. \end{cases} \quad (15)$$

Hence, we can obtain the conditions for each pattern in Figure 1 to prevail.

Lemma 8. The conditions for each pattern in Figure 1 to prevail are characterized as follows.

- (i) if neither condition in (15) holds, pattern 1 arises;
- (ii) if both conditions in (15) hold, and $k_4 < k_1$, pattern 2 arises;
- (iii) if conditions in (14) and (15) hold, and $k_3 < k_1 < k_4$, pattern 3 arises;
- (iv) if conditions in (14) and (15) hold, and $k_1 < k_3$, pattern 4 arises.

Pattern 1 and pattern 2 illustrate the situation that advance selling at the regular selling price is dominated by either a premium or a discount. Thus, advance selling at the regular selling price is no longer an optimal choice for the retailer. However, under pattern 3 and pattern 4, advance selling at the regular selling price dominates the other two strategies when k lies in some certain intervals (consumer valuation uncertainty is high). Thus, we have the following proposition by combining Figure 1 and Lemma 8.

Proposition 3. The conditions for each pricing strategy to prevail can be described as follows

- (i) In the case that either pattern 1 or pattern 2 arises,
 - (a) when $k \in [k_1, 1)$, the optimal strategy is advance selling at a discount price;
 - (b) when $k \in (0, k_1)$, the optimal strategy is advance selling at a premium price.

(ii) In the case that pattern 3 arises,

- (a) when $k \in [k_4, 1)$, the optimal strategy is advance selling at a discount price;
- (b) when $k \in (k_2, k_4)$, the optimal strategy is advance selling at the regular selling price or no advance selling;
- (c) when $k \in (0, k_2]$, the optimal strategy is advance selling at a premium price.

(iii) In the case that pattern 4 arises,

- (a) when $k \in [k_1, k_3] \cup [k_4, 1)$, the optimal strategy is advance selling at a discount price;
- (b) when $k \in (k_3, k_4)$, the optimal strategy is advance selling at the regular selling price or no advance selling;
- (c) when $k \in (0, k_1)$, the optimal strategy is advance selling at a premium price.

5.3 Optimal pricing strategy through numerical analysis

To better demonstrate the pricing decision, numerical examples are conducted to show how the retailer chooses the optimal pricing strategy.

Example 1. This example supports pattern 1. It is constructed with $c = 100$, $s = 50$, $\tau_i = 0.8$, $v_L = 150$, $v_H = 200$, $m_i = 200000$, and $\alpha = 0.5$. In this example, $\eta = 0.13$, $\bar{\alpha} = 0.84 > \alpha$, and $k_1 = 0.43$. Thus, if $k \geq 0.43$, the optimal strategy is (EV, v_H) ; if $k < 0.43$, the optimal strategy is (\hat{p}, v_L) .

Example 2. This example supports pattern 2. It is constructed with $c = 50$, $s = 40$, $\tau_i = 1.8$, $v_L = 100$, $v_H = 140$, $m_i = 200000$, and $\alpha = 0.85$. In this example, $\eta = 0.04$, $\bar{\alpha} = 0.96 > \alpha$, and $k_1 = 0.61$. Thus, if $k \geq 0.61$, the optimal strategy is (EV, v_H) ; if $k < 0.61$, the optimal strategy is (\hat{p}, v_L) .

Example 3. This example supports pattern 3. It is constructed with $c = 100$, $s = 60$, $\tau_i = 0.8$, $v_L = 120$, $v_H = 200$, $m_i = 200000$, and $\alpha = 0.65$. In this example, $\eta = 0.14$, $\bar{\alpha} = 0.70 > \alpha$, $k_1 = 0.26$, $k_2 = 0.25$, $k_3 = 0.21$ and $k_4 = 0.63$. Thus, if $k \geq 0.63$, the optimal strategy is (EV, v_H) ; if $0.25 < k < 0.63$, the optimal strategy is (v_H, v_H) or v_H ; if $k \leq 0.25$, the optimal strategy is (\hat{p}, v_L) .

Example 4. This example supports pattern 4. It is constructed with $c = 30$, $s = 20$, $\tau_i = 0.25$, $v_L = 60$, $v_H = 100$, $m_i = 200000$, and $\alpha = 0.62$. In this example, $\eta = 0.02$, $\bar{\alpha} = 0.89 > \alpha$, $k_1 = 0.42$, $k_2 = 0.44$, $k_3 = 0.55$ and $k_4 = 0.84$. Thus, if $k \geq 0.84$ or $0.42 < k \leq 0.55$, the optimal strategy is (EV, v_H) ; if $0.55 < k < 0.84$, the optimal strategy is (v_H, v_H) or v_H ; if $k \leq 0.42$, the optimal strategy is (\hat{p}, v_L) .

Suppose $k = 0.5$, the retailer's optimal pricing strategy changes when different patterns arise. Under pattern 1, the optimal pricing strategy is advance selling at a discount price; under pattern 2, advance selling at a premium price is the best; under pattern 3, the retailer will either choose advance selling at the regular selling price or no advance selling; and under pattern 4, the optimal pricing strategy is advance selling at a discount price. All four pricing strategies may arise in the equilibrium. With the same consumer valuation uncertainty, if the parameter values in the model change, the optimal pricing strategy changes.

6 The effects of α on the optimal strategy and profit

As defined in Section 4.2, α denotes the proportion of experienced consumers in the market. In this section, how the retailers optimal strategy and profit are affected by α is investigated. Without loss of generosity, consider, for example, pattern 3 arises.

Since α denotes the proportion of experienced consumers in the market, a change in α leads to different consumer compositions, which affects the expected demand in the second period, hence affects the stock-out probability.

Lemma 9. *The stock-out probability, η in (5), decreases in α .*

The intuition for the above result in regard to a change in the α is as follows. The number of experienced consumers is certain. With more experienced consumers purchasing in the second period, the demand uncertainty to the retailer decreases; therefore he can better satisfy the orders in the second period, resulting a lower stock-out probability.

Next, we look at how profit functions change in α . To do that, we can rewrite $m_e = \alpha m$ and $m_i = (1 - \alpha)m$ in these expressions and calculate the derivatives of each profit function with respect to α .

Lemma 10. *The proportion of experienced consumers, α , affects retailer's profit in the way that: $\frac{\partial \Pi_{premium}(k)}{\partial \alpha} < 0$; $\frac{\partial \Pi_{discount}(k)}{\partial \alpha} < 0$; and $\frac{\partial \Pi_{regular}(k)}{\partial \alpha} > 0$.*

The intuition is straightforward. An increase in α changes the composition of experienced and inexperienced consumers in the total consumer population by decreasing the group size of inexperienced consumers and increasing the group size of experienced consumers. Under advance selling price at a premium price, all inexperienced consumers pre-order in the first period at \hat{p} while all experienced consumers buy in the second period at v_L , where $\hat{p} > v_L$. Thus, an increase in α hurts the retailer by decreasing the profit margin selling to additional experienced consumers. Under a pre-order discount, all inexperienced consumers and high type experienced consumers buy in the first period at EV . Thus, an increase in α hurts the retailer buy decreasing the overall demand. However, such a change in α reduces the overall consumer valuation uncertainty and demand uncertainty, which is of great benefit to the retailer when he adopts advance selling at the regular selling price or sells in the regular selling season without advance selling.

For the rest analysis in this section, we focus on small changes in α so that the pattern of profit functions does not jump to one of the other three patterns. To better illustrate how the optimal pricing strategy switches and examine how the optimal profit changes with α , Figure 2 depicts the profit functions under α_1 and α_2 , assuming that $\alpha_1 < \alpha_2$.

In Figure 2, the solid lines denote profit functions under $\alpha = \alpha_1$ and k_2, k_4 denote the threshold values of k ; the dash lines denote profit functions under α_2 and k'_2, k'_4 denote the threshold values of k in this case. Furthermore, to examine how the optimal profit is affected, we use k_2^{mix} to denote the intersection of $\Pi_{premium}(k) |_{\alpha=\alpha_1}$ and $\Pi_{regular}(k) |_{\alpha=\alpha_2}$, and k_4^{mix} to denote the upper intersection of $\Pi_{discount}(k) |_{\alpha=\alpha_1}$ and $\Pi_{regular}(k) |_{\alpha=\alpha_2}$. Lemma 10 implies that in Figure 2, $\Pi_{premium}(k)$ and $\Pi_{discount}(k)$ both move down while $\Pi_{regular}(k)$ moves up when α increases. Hence, it follows that k_2 decreases and k_4 increases.

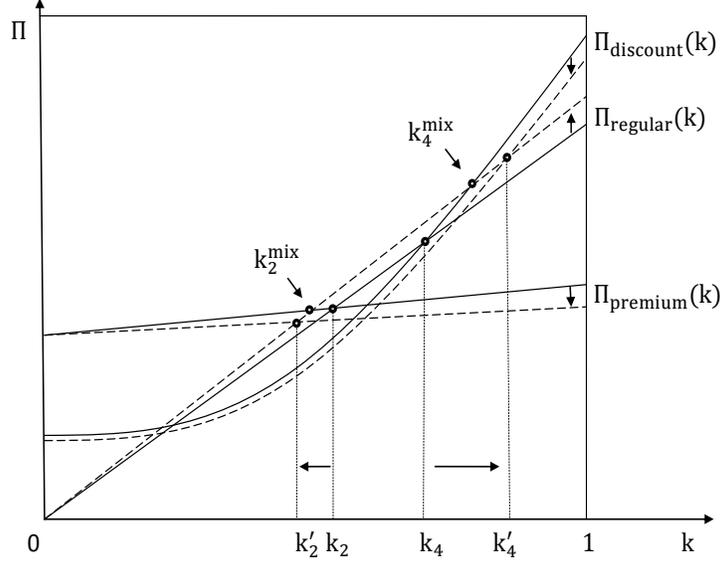


Figure 2: How does α affect the optimal strategy and profit under pattern 3

There are four possible optimal pricing strategies under pattern 3: advance selling at a premium price, advance selling at a discount price, advance selling at the regular selling price and no advance selling. As depicted in Figure 2, (k_2, k_4) expands at both sides. It implies that advance selling at the regular selling price, or, equivalently, no advance selling, is increasingly beneficial to the retailer because an increasing in the group size of experienced consumers helps to reduce the demand uncertainty. Specifically, the retailer will switch from advance selling at a premium price to advance selling at the regular selling price or no advance selling if $k \in (k_2', k_2]$, and switch from advance selling at a discount price to advance selling at the regular selling price or no advance selling if $k \in [k_4, k_4')$; for any other value of k , the optimal pricing strategy remains the same. With regard to the optimal profit, we can see from the figure that it increases if $k \in (k_2^{mix}, k_4^{mix})$ and decreases otherwise.

Proposition 4. *Under pattern 3, with a small increase in α , the retailer's optimal pricing strategy is as depicted in Figure 2. In particular, the retailer becomes more willing to choose advance selling at the regular selling price, or no advance selling; the optimal profit increases when $k \in (k_2^{mix}, k_4^{mix})$; otherwise, it decreases.*

Similar analysis applies to the other three patterns. Since intersections change as α increases, there always exists optimal strategy switching. As long as advance selling at the regular selling price or no advance selling become one of the possible optimal strategies (under pattern 3 and pattern 4), the retailer will be more willing to switch from either advance selling at a premium price or advance selling at a discount price to one of them. With regard to the optimal profit, under pattern 1 and pattern 2, it always decreases because both $\Pi_{premium}(k)$ and $\Pi_{discount}(k)$ decrease; under pattern 4, it can either decrease or increase.

7 Extensions

In this section, the previous analyses are extended in two directions. Section 7.1 considers there exists uncertainty in consumer valuation distribution (k is unknown), while Section 7.2 examines the situation with no experienced consumers.

7.1 Uncertainty in consumer valuation distribution

What happens if k is unknown in the model while we keep other settings in Section 3 unchanged? Particularly, assume that k follows a two-point distribution: $\text{Prob}(k_H) = \gamma$ and $\text{Prob}(k_L) = 1 - \gamma$, where $k_L < k_H$ and $\gamma \in (0, 1)$. It implies that inexperienced consumers are not sure about the probability that their valuations will be high. In addition, the retailer is uncertain about the proportion of high type consumers in the market. Therefore, the second-period demand uncertainty for the retailer not only comes from the number of inexperienced consumers, but also from the proportion of high type consumers in the market.

To inexperienced consumers, the expected valuation of this product in the advance selling season can be described as $EV = E[k]v_H + (1 - E[k])v_L$. Following the discussion in Section 4, if the retailer decides to adopt advance selling, he will choose from the following advance selling strategies: (\hat{p}, v_L) , (\bar{p}, v_L) , (EV, v_H) , and (v_H, v_H) .

As before, the retailer will not sell at v_L without advance selling. Note that selling at v_H generates the same amount of profit as that from advance selling at (v_H, v_H) in Section 5. Will the retailer sell at v_H without advance selling when k is unknown?

First, look at the case when the retailer sells in the regular selling season at v_H without advance selling. Since there is no advance selling season, $D_1 = 0$. High type experienced consumers and high type inexperienced consumers will purchase in the regular selling season. The total demand from consumers is

$$D_2 = \begin{cases} k_H m_e + k_H M_i, & \text{with probability } \gamma, \\ k_L m_e + k_L M_i, & \text{with probability } 1-\gamma. \end{cases}$$

As a result, in addition to the random demand from inexperienced consumers, the retailer does not know the exact demand from experienced although he knows the number of experienced consumers. He produces according to the expected total demand described above at the beginning of the regular selling season and consumers with valuation v_H make purchases.

Second, in the case that the retailer adopts advance selling at (v_H, v_H) , high type experienced consumers pre-order and high type inexperienced consumers purchase in the second period. Let D_1 denote the number of pre-orders from high type experienced consumers. Since $D_1 = k m_e$, the retailer infers k from pre-orders. Suppose he infers $k = k_L$. Therefore, the retailer forecast the demand from inexperienced consumers will be $k_L M_i$, which follows a log-normal distribution. Thus, the retailer produces $D_1 + q^*$ to satisfy the total demand, where q^* is the optimal quantity for the random demand from inexperienced consumers.

Compared with no advance selling, the demand uncertainty under advance selling at (v_H, v_H) decreases while the total demand from consumers stay the same. Then we can conclude that advance selling at (v_H, v_H) is superior to selling at v_H without advance selling. Since the optimal

advance selling strategy generates at least the same amount of profit as (v_H, v_H) does, we have the following proposition.

Proposition 5. *When k is unknown, advance selling is superior to no advance selling. The retailer will always choose advance selling, either at a discount price, at a premium price, or at the regular selling price.*

When consumer valuation distribution is uncertain, as discussed above, the pre-order information provides the retailer with the opportunity to learn the distribution of consumer valuation and then better forecast the random demand in the second period.

Corollary 1. *When there exists uncertainty concerning consumer valuation distribution, the retailer can learn some information from pre-orders, which helps him better forecast the future demand and thus improve total expected profit.*

This result shows that if the demand uncertainty in the regular selling season not only comes from the group size of inexperienced consumers but also from consumer valuation distribution, learning from pre-orders benefits the retailer because it helps to better forecast the demand in the regular selling season. In such a context, the retailer will always adopt advance selling (Loginova, Wang and Zeng 2012). However, as discussed in Section 5.1 when consumer valuation distribution is known to the market, advance selling is not always optimal.

7.2 With no experienced consumers

Consider a retailer releases a completely new product to the market.⁸ There are no experienced consumers. That is, $m_e = 0$ in the model. All consumers are inexperienced consumers and they are uncertain about their valuations in the first period. Under the same model setup as the one in previous sections, we are going to study the retailer's optimal pricing strategy when $m_e = 0$ so as to better understand the role played by experienced consumers in the model.

Let Π_m denote the expected total profit for the retailer when there are no experienced consumers. As noted before, we use backward induction; the optimal regular selling price should be either v_L or v_H . When $p_2^* = v_L$, consumers pre-order if and only if $p_1 \leq \hat{p}$; otherwise, all wait to buy in the second period. Therefore, if the retailer decides to adopt advance selling, he will set $p_1^* = \hat{p}$ to make all consumers pre-order and get the highest profit margin. The resulting expected total profit from (\hat{p}, v_L) is

$$\Pi_m(\hat{p}, v_L) = (\hat{p} - c)m_i. \quad (16)$$

When $p_2^* = v_H$, consumers pre-order if and only if $p_1 \leq EV$. As a result, the retailer will either set $p_1 = EV$ which results in an expected total profit

$$\Pi_m(EV, v_H) = (EV - c)m_i, \quad (17)$$

or charges $p_1 = v_H$ ⁹ with an expected total profit described by

$$\Pi_m(v_H, v_H) = \pi_H. \quad (18)$$

⁸It includes the first generation of any series product.

⁹At any $p_1 \in (EV, v_H]$, the retailer generates the same amount of expected total profit. I assume that the retailer sets $p_1 = v_H$ so as to get the highest profit margin though there are no pre-order.

Lemma 11. *Without experienced consumers, advance selling at a premium price, (\hat{p}, v_L) , is strictly dominated by advance selling at a discount price, (EV, v_H) . The optimal pricing strategy is either advance selling at a discount, advance selling at the regular selling price, or no advance selling.*

Next, we are going to look at the situation that the retailer does not adopt advance selling. As before, he will always charges either v_L or v_H in the regular selling season. Following the discussion in Section 5, selling in the regular selling season at v_L is strictly dominated by advance selling, while selling at v_H generates the same amount of profit as (v_H, v_H) does.

After considering both advance selling and no advance selling, we have the following Proposition on the retailer's optimal pricing strategy.

Proposition 6. *There exists a threshold $k_5 = \frac{v_L - c}{(v_H - s)(1 - \Phi(\tau_i - z_\beta)) - (v_H - v_L)}$. If $0 < k_5 < 1$, when $k > k_5$, the retailer will choose advance selling at (v_H, v_H) or sell at v_H in the regular selling season without advance selling. Otherwise, the retailer will always choose advance selling at a discount, that is, (EV, v_H) .*

Proposition 6 implies that there are three possible pricing strategies for a completely new product: advance selling at a discount price, advance selling at the regular selling price and no advance selling.

Corollary 2. *There are no incentives for the retailer to charge advance selling price premium when there are no experienced consumers in the market.*

Nasiry and Popescu (2012) explain why firms charge high prices in advance through consumers' regret. However, this result provides an alternative explanation for advance selling price premium when consumers are heterogeneous in their valuations. For new products with early generation(s), the presence of experienced consumers provides retailers with the incentives to charge a premium price in advance.

8 Conclusion

This paper studies advance selling for a retailer who faces a group of experienced consumers who have prior experience with early generation(s) of this product while other consumers do not. Rather than take the regular selling price as given in the literature, this paper allows the retailer to determine the regular selling price as well as the advance selling price and studies his optimal pricing strategy. Besides, by further examining a model without experienced consumers, this paper provides a new explanation for why the retailer charges a premium price in the advance selling season. Furthermore, it investigates the situation when there exists uncertainty in consumer valuation distribution in the extension.

Main results of this paper are summarized below.

For a new product with early generation(s), the retailer has four possible pricing strategies: advance selling at a premium price, advance selling at a discount price, advance selling at the regular selling price, and no advance selling.

- Under advance selling at a premium price, the optimal prices in both periods are such that all inexperienced consumers pre-order and all experienced consumers wait to buy in the selling season.
- Under advance selling at a discount price, the optimal prices in both periods are such that inexperienced consumers and high type experienced consumers pre-order while low type inexperienced consumer do not buy this product. Compared to advance selling at a premium price, both prices are higher.
- Advance selling at the regular selling price generated the same amount of expected profit as no advance selling. Under both pricing strategies, only high type consumers buy this product. Also, the demand uncertainty in the regular selling season remains for the retailer. The only difference is the shopping timing for high experienced consumers. They will buy in advance if pre-orders are available.

For a completely new product with no experienced consumers in the market, the retailer has no incentives to adopt advance selling at a premium price. There are three possible pricing strategies for pre-orders: advance selling at a discount price, advance selling at the regular selling price, and no advance selling. Without experienced consumers, there are no incentives for the retailer to implement advance at a premium price.

- To both consumers and the retailer, advance selling at the regular selling price is exactly the same as no advance selling.
- Under advance selling at a discount price, all consumers pre-order at the advance selling price.

If the probability for a consumer to realize a high valuation is uncertain (i.e. k is uncertain) in the model, the retailer will always choose to adopt advance selling. The optimal advance selling strategy may be either at a premium price, at a discount price, or at the regular selling price. In this extended model, the retailer will be able to learn the value of k and thus better forecast the future demand.

For future research, one possible direction is to introduce competition in the model and study the optimal advance selling decisions of the retailers. Another direction is to study a different information structure of advance selling. For example, consumers arrive in the advance selling season at different times and the popularity of the product could work as a signal for them to update their valuations. For series products, given the existence of the old generation when a new generation comes out, it would be interesting to study the pricing strategies for both generations in a two-period model.

Appendix

Proof of Lemma 2:

First, $\hat{p} - v_L = EV - v_L - (1 - \eta)(EV - v_L) = \eta(EV - v_L) > 0$. Second, $\bar{p} - \hat{p} = v_H - (1 - \eta)(v_H - v_L) - EV + (1 - \eta)(EV - v_L) = v_H - EV - (1 - \eta)(v_H - v_L - EV + v_L) = \eta(v_H - EV) > 0$. Third, it is obvious from (4) that $\bar{p} < v_H$.

Proof of Lemma 3:

The retailer will adopt advance selling at (\bar{p}, v_L) if and only if $\Pi(\bar{p}, v_L) \geq \Pi(\hat{p}, v_L)$. That is, $\eta(v_H - v_L)m_e k + (v_L - c)m_e + \pi_L^* \geq \eta(v_H - v_L)m_i k + (v_L - c)(m_i + m_e)$, where $k \in (0, 1)$. After simplification, $\eta(v_H - v_L)(m_e - m_i) > (v_L - c)m_i - \pi_L^*$ is needed for $\Pi(\bar{p}, v_L) \geq \Pi(\hat{p}, v_L)$ to hold. In addition, we have that $m_e = \alpha m$, $m_i = (1 - \alpha)m$ and $\pi_L^* = (v_L - s)(1 - \Phi(\tau_i - z_\beta))m_i$. Then we can rewrite the above inequality as $(2\alpha - 1)\eta(v_H - v_L) > \{v_L - c - (v_L - s)(1 - \Phi(\tau_i - z_\beta))\}(1 - \alpha)$. By solving this inequality, we have $\alpha > \frac{\eta(v_H - v_L) + v_L - c - (v_L - s)(1 - \Phi(\tau_i - z_\beta))}{2\eta(v_H - v_L) + v_L - c - (v_L - s)(1 - \Phi(\tau_i - z_\beta))} \equiv \bar{\alpha}$. In other words, we need $\alpha > \bar{\alpha}$ so as to have $\Pi(\bar{p}, v_L) \geq \Pi(\hat{p}, v_L)$.

Derivation of (11), (12) and (13):

From (3) and (6), $\Pi_{premium}(k) = \Pi(\hat{p}, v_L) = (v_L - c)m_e + \{kv_H + (1 - k)v_L - (1 - \eta)(kv_H + (1 - k)v_L - v_L) - c\}m_i = (v_L - c)m_e + \{\eta(v_H - v_L)k + v_L - c\}m_i = \eta(v_H - v_L)m_i k + (v_L - c)(m_e + m_i)$. From (9), $\Pi_{discount}(k) = \Pi(EV, v_H) = (kv_H + (1 - k)v_L - c)km_e + (kv_H + (1 - k)v_L - c)m_i = (v_H - v_L)m_e k^2 + \{(v_H - v_L)m_i + (v_L - c)m_e\}k + (v_L - c)m_i$. $\Pi_{same}(k) = \{(v_H - c)m_e + (v_H - s)(1 - \Phi(\tau_i - z_\beta))m_i\}k$ is directly from (10).

Proof of Lemma 5:

(i) is obvious from the fact that the coefficients of k and k^2 are positive. Regarding (ii), we have $\lim_{k \rightarrow 0} \Pi_{premium}(k) = (v_L - c)(m_e + m_i)$; $\lim_{k \rightarrow 0} \Pi_{discount}(k) = (v_L - c)m_i$; and $\lim_{k \rightarrow 0} \Pi_{regular}(k) = 0$. Thus, (ii) is obtained. Regarding (iii), we have $\lim_{k \rightarrow 1} \Pi_{premium}(k) = \eta(v_H - v_L)m_i + (v_L - c)(m_e + m_i)$; $\lim_{k \rightarrow 1} \Pi_{discount}(k) = (v_H - v_L)m_e + (v_H - v_L)m_i + (v_L - c)m_e + (v_L - c)m_i = (v_H - c)(m_e + m_i)$; and $\lim_{k \rightarrow 1} \Pi_{regular}(k) = (v_H - c)m_e + (v_H - s)(1 - \Phi(\tau_i - z_\beta))m_i$.

Thus, $\lim_{k \rightarrow 1} \Pi_{discount}(k) - \lim_{k \rightarrow 1} \Pi_{premium}(k) = (v_H - v_L)(m_e + m_i) - \eta(v_H - v_L)m_i > 0$. For $\lim_{k \rightarrow 1} \Pi_{discount}(k) > \lim_{k \rightarrow 1} \Pi_{regular}(k)$, we need to show that $v_H - c > (v_H - s)(1 - \Phi(\tau_i - z_\beta))$, which is directly from the fact that $\Phi(z_\beta) > \Phi(z_\beta - \tau_i)$.

Derivation of k_1 :

The intersection, k_1 , satisfies the following equation: $\Pi_{premium}(k) = \Pi_{discount}(k)$. After simplification, that is, $(v_H - v_L)m_e k^2 + \zeta k - (v_L - c)m_e = 0$. Since $k_1 > 0$, we have $k_1 = \frac{\sqrt{\zeta^2 + 4(v_H - v_L)(v_L - c)m_e^2} - \zeta}{2(v_H - v_L)m_e}$.

Derivation of k_2 :

The value of k_2 comes straightforwardly from $\Pi_{premium}(k_2) = \Pi_{regular}(k_2)$.

Derivation of k_3 and k_4 :

The intersections, k_3 and k_4 , satisfy the following equation: $\Pi_{discount}(k) = \Pi_{regular}(k)$. After simplification, that is, $(v_H - v_L)m_e k^2 + \xi k + (v_L - c)m_i = 0$. Thus we get the values for k_3

and k_4 as in Section 5.2 if $\xi^2 - 4(v_H - v_L)m_e(v_L - c)m_i > 0$.

Proof of Lemma 6:

We know that $\lim_{k \rightarrow 0} \Pi_{premium}(k) > \lim_{k \rightarrow 0} \Pi_{discount}(k)$. Then if $\lim_{k \rightarrow 1} \Pi_{regular}(k) > \lim_{k \rightarrow 1} \Pi_{premium}(k)$, we will have $0 < k_2 < 1$. Equation (14) is obtained by rewriting $\lim_{k \rightarrow 1} \Pi_{regular}(k) > \lim_{k \rightarrow 1} \Pi_{premium}(k)$.

Proof of Lemma 7:

We know that k_3 and k_4 satisfy $(v_H - v_L)m_e k^2 + \xi k + (v_L - c)m_i = 0$. Thus $\xi^2 - 4(v_H - v_L)m_e(v_L - c)m_i > 0$ ensures that there are two intersections between $\Pi_{premium}(k)$ and $\Pi_{discount}(k)$. Moreover, because $(v_H - v_L)m_e k^2 + \xi k + (v_L - c)m_i > 0$ when $k = 0$ and $k = 1$, with $\xi^2 - 4(v_H - v_L)m_e(v_L - c)m_i > 0$, $0 < -\frac{\xi}{2(v_H - v_L)m_e} < 1$ ensures $k_3, k_4 \in (0, 1)$.

Proof of Lemma 9:

We can rewrite η as $\eta(\alpha) = E[\left(\frac{M_i(\alpha) - q_L^*}{\alpha m + M_i(\alpha)}\right)^+]$, where $M_i(\alpha) \sim \text{LN}\left(\ln(1 - \alpha)m - \frac{\tau_i^2}{2}, \tau_i^2\right)$. $\forall 0 < \alpha_1 < \alpha_2 < 1$, $M_i(\alpha_2) = \frac{1 - \alpha_2}{1 - \alpha_1} M_i(\alpha_1)$. Let $I(\alpha) = \frac{M_i(\alpha) - q_L^*}{\alpha m + M_i(\alpha)}$, we have $I(\alpha_2) - I(\alpha_1) = \frac{M_i(\alpha_2) - q_L^*}{\alpha_2 m + M_i(\alpha_2)} - \frac{M_i(\alpha_1) - q_L^*}{\alpha_1 m + M_i(\alpha_1)} = \frac{\frac{1 - \alpha_2}{1 - \alpha_1} M_i(\alpha_1) - q_L^*}{\alpha_2 m + \frac{1 - \alpha_2}{1 - \alpha_1} M_i(\alpha_1)} - \frac{M_i(\alpha_1) - q_L^*}{\alpha_1 m + M_i(\alpha_1)} = \frac{\alpha_1 m + q_L^*}{\alpha_1 m + M_i(\alpha_1)} - \frac{\alpha_2 m + q_L^*}{\alpha_2 m + \frac{1 - \alpha_2}{1 - \alpha_1} M_i(\alpha_1)} < \frac{(\alpha_2 - \alpha_1)m + \alpha_1 m + q_L^*}{(\alpha_2 - \alpha_1)m + \alpha_1 m + M_i(\alpha_1)} - \frac{\alpha_2 m + q_L^*}{\alpha_2 m + \frac{1 - \alpha_2}{1 - \alpha_1} M_i(\alpha_1)} < 0$. That is, $I(\alpha_2) < I(\alpha_1)$. Then we have $\eta(\alpha_2) < \eta(\alpha_1)$.

Proof of Lemma 10:

First, $\frac{\partial \Pi_{premium}(k)}{\partial \alpha} = \frac{\partial \eta(v_H - v_L)(1 - \alpha)mk}{\partial \alpha} = \frac{\partial \eta}{\partial \alpha}(v_H - v_L)(1 - \alpha)mk - \eta(v_H - v_L)mk < 0$; second, $\frac{\partial \Pi_{discount}(k)}{\partial \alpha} = (EV - c)km - (EV - c)m < 0$; last, $\frac{\partial \Pi_{regular}(k)}{\partial \alpha} = (v_H - c)km - k(v_H - s)(1 - \Phi(\tau_i - z_\beta))m = [(v_H - c) - (v_H - s)(1 - \Phi(\tau_i - z_\beta))]km > 0$.

Proof of Lemma 11:

It comes straightforwardly from the fact that $\hat{p} < EV$.

Proof of Proposition 6:

We can rewrite (17) as $\Pi_m(EV, v_H) = (v_H - v_L)m_i k + (v_L - c)m_i$; (18) as $\Pi_m(v_H, v_H) = (v_H - s)(1 - \Phi(\tau_i - z_\beta))m_i k$. Let k_5 be the solution for $\Pi_m(EV, v_H) = \Pi_m(v_H, v_H)$. We have $k_5 = \frac{v_L - c}{(v_H - s)(1 - \Phi(\tau_i - z_\beta)) - (v_H - v_L)}$. If $0 < k_5 < 1$, when $k > k_5$, we have $\Pi_m(EV, v_H) < \Pi_m(v_H, v_H)$; when $k \leq k_5$, we have $\Pi_m(EV, v_H) \geq \Pi_m(v_H, v_H)$. Moreover, if $k_5 \leq 0$ or $k_5 \geq 1$, $\Pi_m(EV, v_H) > \Pi_m(v_H, v_H)$. Thus, we have Proposition 6.

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